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Mathematical Systems Theory I

Modelling, State Space Analysis,
Stability and Robustness

With 180 Figures

 Springer

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